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CP violation in radiative Z decays

D. Bruß[1], O. Nachtmann[2], P. Overmann[2]

[1] Physics Department, University of Oxford, Clarendon Laboratory, Parks Road, Oxford OX1 3PU, UK
[2] Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg, Germany

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Abstract. We propose to test the CP symmetry in the reactions $\mathrm{Z} \to \mu^+\mu^-\gamma$ and $\mathrm{Z} \to \tau^+\tau^-\gamma$. The experimental analysis of angular correlations allows to determine a set of effective couplings: the electric and weak dipole moments of the muon and the tau lepton, and in particular chirality conserving 4-particle couplings, all of which can be induced by CP–violation in renormalizable theories of electroweak interactions beyond the Standard Model. We update an indirect bound on the weak dipole moment of the muon.

1 Introduction

The search for CP violation in Z boson decays provides an interesting tool to investigate possible new physics beyond the Standard Model. Several proposals have been made how to look for CP violation in processes such as $\mathrm{Z} \to \ell^+\ell^-$, $\ell^+\ell^-\gamma$, $\mathrm{q\bar{q}}$, $\mathrm{q\bar{q}}\gamma$, $\mathrm{q\bar{q}G}$, where $\ell = \mathrm{e}, \mu, \tau$ leptons, q = u,d,s,c,b quarks, and G = gluon (cf. [1–12] and references therein). CP–violating effects in the above reactions can be parametrized by form factors, among them the well known electric and weak dipole moments of quarks and leptons, or one can use the effective Lagrangian approach. Experimental investigations of CP–odd effects in Z decays have been performed resulting in upper limits on the weak dipole moment of the τ lepton [13–15] and on a CP violation parameter in $\mathrm{Z} \to \mathrm{b\bar{b}G}$ [16]. An "indirect" limit on the τ electric dipole moment was given in [17].

Today, a large number of Z bosons has been produced in electron–positron collisions both at LEP (CERN) and at SLC (SLAC). Thus investigations of relatively rare events like radiative three–body decays are possible. In this article we investigate in detail the unpolarized electron–positron annihilation to a muon or tau lepton pair and a hard photon with respect to CP violation:

$$\mathrm{e}^+(p_+) + \mathrm{e}^-(p_-) \to \mathrm{Z} \to \ell^+(k_{\bar{\ell}}) + \ell^-(k_\ell) + \gamma(q), \quad (\ell = \mu, \tau). \quad (1)$$

Within the framework of the Standard Model (SM), CP violation in these leptonic reactions occurs only at higher orders in perturbation theory, leading to unobservably small effects. However, interactions beyond the SM such as the exchange of virtual photinos or other neutralinos, neutral Higgs bosons, excited leptons, or leptoquarks, can induce CP–violating contributions to the interaction of leptons and gauge bosons. A calculation of the dipole form factors of the τ lepton in several extensions of the SM can be found in [18]. Most of these models have in common that they lead to CP–violating effective couplings, typically generated at one–loop level, which are are not flavour–universal but can grow substantially with the mass of the quarks and leptons involved.

In the following section a model–independent parametrization of CP–odd couplings relevant for the reactions (1) is discussed. Then we propose some easily measurable CP–odd momentum correlations which are optimized to extract information on these couplings from the data. In Sects. 4 and 5 we discuss in some detail the level of accuracy of CP tests obtainable in Z decays to $\mu^+\mu^-\gamma$ and $\tau^+\tau^-\gamma$, respectively.

2 Effective Lagrangian approach

In order to describe in a model–independent way possible CP–violating effects we use an effective Lagrangian with CP–odd operators of mass dimensions $d > 4$. Restricting ourselves to $d \leq 6$ (*after* symmetry breaking!), the operators which are relevant for the reactions considered here are given by (c.f. [2,6])

$$\begin{aligned} \mathcal{L}_{\mathrm{CP}}(x) = & \qquad\qquad\qquad\qquad\qquad\qquad (2) \\ & \sum_\ell \Big\{ -\frac{i}{2}\tilde{d}_\ell^\gamma \;\bar{\ell}(x)\sigma^{\mu\nu}\gamma_5\ell(x)\left[\partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)\right] \\ & -\frac{i}{2}\tilde{d}_\ell^{\mathrm{Z}} \;\bar{\ell}(x)\sigma^{\mu\nu}\gamma_5\ell(x)\left[\partial_\mu Z_\nu(x) - \partial_\nu Z_\mu(x)\right] \\ & +\left[f_{\mathrm{V}\ell}\bar{\ell}(x)\gamma^\nu\ell(x) + f_{\mathrm{A}\ell}\bar{\ell}(x)\gamma^\nu\gamma_5\ell(x)\right] Z^\mu(x) \\ & \times\left[\partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)\right]\Big\}, \end{aligned}$$

where $\tilde{d}_\ell^\gamma, \tilde{d}_\ell^Z$ are the electric dipole moment (EDM) and weak dipole moment (WDM) coupling parameters of the leptons $\ell = \mu, \tau$ and $f_{V\ell}, f_{A\ell}$ are real coupling constants corresponding to CP–violating Z–photon–lepton 4-particle vertices. The photon and Z–boson fields are denoted by A_μ and Z_μ, respectively. The authors of [11] have given a form factor decomposition for reactions of the type (1). In such a framework our couplings in (2) correspond in essence to the CP–violating form factors with minimal momentum dependence.

Following previous analyses [2,8] we introduce the dimensionless parameters $\hat{f}_{V\ell}, \hat{f}_{A\ell}, \hat{\tilde{d}}^{\gamma}_{\ell}, \hat{\tilde{d}}^{Z}_{\ell}$ defined by

$$\begin{aligned} f_{V\ell,A\ell} &= -\frac{e^2 \cdot Q_\ell}{\sin\theta_W \cos\theta_W m_Z^2} \hat{f}_{V\ell,A\ell} \\ &= \frac{2.62 \cdot 10^{-5}}{\mathrm{GeV}^2} \hat{f}_{V\ell,A\ell}, \qquad (3) \\ \tilde{d}^{\gamma,Z}_{\ell} &= -\frac{e \cdot Q_\ell}{\sin\theta_W \cos\theta_W m_Z} \hat{\tilde{d}}^{\gamma,Z}_{\ell} \\ &= 5.15 \cdot 10^{-16} e\mathrm{cm} \cdot \hat{\tilde{d}}^{\gamma,Z}_{\ell}, \qquad (4) \end{aligned}$$

where $\ell = \mu, \tau$ and $Q_\ell = -1$, and $e^2/(4\pi)$ is the fine structure constant. We use $m_Z = 91.19$ GeV and $\sin^2\theta_W = 0.23$ for the numerics. We work to leading order in the CP–violating couplings of $\mathcal{L}_{CP}$ and in the couplings of the SM. Then the parameters in the effective Lagrangian are identical to the corresponding form factors, and imaginary parts of the form factors are absent. This point was discussed in detail in [12], and it was shown that in this order one has

$$\begin{aligned} \mathrm{Re}\; d^{\gamma,Z}_{\ell}(s) &= \tilde{d}^{\gamma,Z}_{\ell}, \qquad (5) \\ \mathrm{Im}\; d^{\gamma,Z}_{\ell}(s) &= 0. \qquad (6) \end{aligned}$$

Here $d^{\gamma,Z}_{\ell}(s)$ are the EDM and WDM *form factors* measurable in the reaction $e^+e^- \to \ell^+\ell^-$ at c.m. energy $\sqrt{s}$. We note that in a pure form factor approach the form factors relevant for $e^+e^- \to \ell^+\ell^-$ and $e^+e^- \to \ell^+\ell^-\gamma$ are *not* directly related. Such a relation is only achieved after making an assumption for the off–shell dependence of the dipole moment form factors. On the other hand, in the effective Lagrangian approach there is no problem to relate the parameters of the above two reactions in leading order. Also in higher orders it is in principle straightforward to work out this relation (cf. [12] for further remarks).

The next step is to calculate the differential cross sections for $e^+e^- \to \ell^+\ell^-$ and $e^+e^- \to \ell^+\ell^-\gamma$ taking into account the SM diagrams (one is shown in Fig. 1a) and the diagrams induced by the couplings of $\mathcal{L}_{CP}$ (Figs. 1b,c,d). Neglecting radiative decays with more than one γ the sum of the two channels above gives the inclusive width

$$\Gamma(Z \to \ell^+\ell^- X) = \Gamma_{SM}(Z \to \ell^+\ell^- X) + \Delta\Gamma_{CP}, \qquad (7)$$

where Γ_{SM} is the SM width and $\Delta\Gamma_{CP}$ is of second order in the CP-violating couplings of $\mathcal{L}_{CP}$. We use this calculation to make an *indirect* estimate of the order of magnitude which is allowed for the CP–violating parameters in $\mathcal{L}_{CP}$ (cf. (2)). The measured widths $\Gamma_{\mu\mu}$ and $\Gamma_{\tau\tau}$ which include all radiative events[1] agree well with the predictions from the SM (see Table 1).

[1] We thank N. Wermes and M. Wunsch for providing information concerning this point

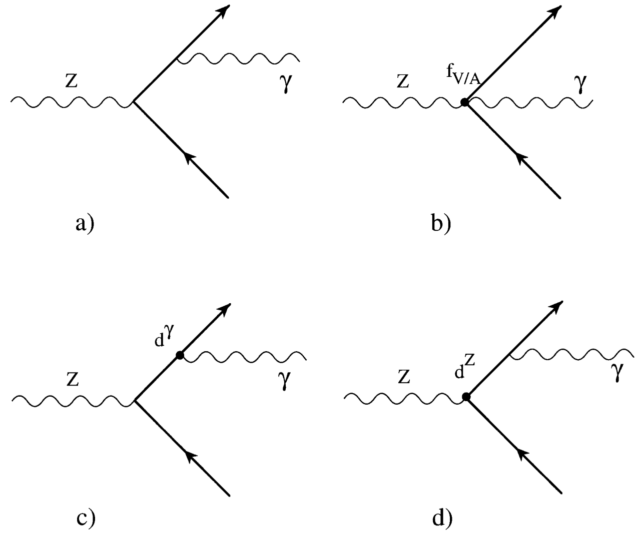

Fig. 1. Feynman diagrams contributing to $Z \to \ell^+\ell^-\gamma$: **a** One of the lowest order Standard Model diagrams **b** Contribution of $f_{A\ell}$ and $f_{V\ell}$ **c** Contribution of the electric dipole moment $\tilde{d}^{\gamma}_{\ell}$ **d** Contribution of the weak dipole moment $\tilde{d}^{Z}_{\ell}$

We define values $\Delta\Gamma_{exp}$ for the "allowed" extra contribution to the width at λ standard deviations as

$$\Delta\Gamma^{\lambda \mathrm{s.d.}}_{exp} = \Gamma_{exp} - \Gamma_{SM} + \lambda \cdot \sqrt{\delta\Gamma^2_{exp} + \delta\Gamma^2_{SM}}. \qquad (8)$$

In principle a combined estimate of the four CP–violation parameters of (2) for each lepton ℓ is possible by ascribing to them any deviation from the width as predicted by the SM, i.e. by setting $\Delta\Gamma_{CP} \le \Delta\Gamma_{exp}$. However, this would ignore the possibility of other non–standard but CP–conserving effects to the widths which can lead to an increase or decrease of the widths as compared to the SM. Thus such a procedure only gives an order of magnitude estimate of the values allowed for the couplings in $\mathcal{L}_{CP}$. We give a simplified estimate by ignoring interference terms induced by the new couplings, i.e. we assume that only $f_{V\ell}$ and $f_{A\ell}$ or $\tilde{d}^{\gamma}_{\ell}$ or $\tilde{d}^{Z}_{\ell}$ contribute in the width $\Gamma(Z \to \ell^+\ell^- X)$. As compared to the other couplings, the contribution of the electric dipole moment $\tilde{d}^{\gamma}_{\ell}$ is suppressed by a factor of α_{ew} in the electroweak perturbation series. Thus the leading terms in $\Delta\Gamma_{CP}$, calculated without phase space cuts and neglecting the lepton masses, are easily obtained from Table 1 of [8] as follows

$$\Delta\Gamma_{CP}(\tilde{d}^{Z}_{\ell}) = \left(\frac{|\tilde{d}^{Z}_{\ell}|}{10^{-17} \cdot e\mathrm{cm}}\right)^2 \cdot 0.24 \text{ MeV}, \qquad (9)$$

$$\Delta\Gamma_{CP}(\hat{f}_{V\ell}, \hat{f}_{A\ell}) = \left(\hat{f}^2_{V\ell} + \hat{f}^2_{A\ell}\right) \cdot 0.042 \text{ MeV}. \qquad (10)$$

Now we can derive λ s.d. limits on $\tilde{d}^{Z}_{\ell}$ and $\hat{f}_{V\ell}, \hat{f}_{A\ell}$ by requiring the theoretical widths (9, 10) to be smaller or equal $\Delta\Gamma^{\lambda \mathrm{s.d.}}_{exp}$ (8):

$$\Delta\Gamma_{CP}(\tilde{d}^{Z}_{\ell}) \le \Delta\Gamma^{\lambda \mathrm{s.d.}}_{exp}, \qquad (11)$$

$$\Delta\Gamma_{CP}(\hat{f}_{V\ell}, \hat{f}_{A\ell}) \le \Delta\Gamma^{\lambda \mathrm{s.d.}}_{exp}, \qquad (12)$$

Table 1. Experimental and theoretical (SM) values of the total decay widths for $\mathrm{Z} \to \mu^+\mu^-\mathrm{X}$ and $\mathrm{Z} \to \tau^+\tau^-\mathrm{X}$ (cf. [19,20])

	Γ_{exp}	Γ_{SM}	$\sqrt{\delta\Gamma^2_{\mathrm{exp}} + \delta\Gamma^2_{\mathrm{SM}}}$
$\mu^+\mu^- X$	83.79 ± 0.22 MeV	83.97 ± 0.01 ± 0.06 MeV	0.23 MeV
$\tau^+\tau^- X$	83.72 ± 0.26 MeV	83.97 ± 0.01 ± 0.06 MeV	0.27 MeV

We see from Table 1 that the differences of the central values, $\Gamma_{\mathrm{exp}} - \Gamma_{\mathrm{SM}}$ are negative. Correspondingly the 1 s.d. values $\Delta\Gamma^{1s.d.}_{\mathrm{exp}}$ are very small. Limits on $\tilde{d}^{\mathrm{Z}}_{\ell}, \hat{f}_{\mathrm{V}\ell}$ and $\hat{f}_{\mathrm{A}\ell}$ obtained from (11, 12) at 1 s.d. are then not to be trusted. As explained above, we should consider (11, 12) not as strict inequalities but only as indications for the order of magnitude which is possible for the CP–violating couplings. We thus prefer to quote the 2 s.d. limits below, since then $\Delta\Gamma^{2s.d.}_{\mathrm{exp}}$ is at least of the order of the combined experimental and theoretical errors given in the last column of Table 1.

In this way we get from (11) for the WDM of the muon the 2 s.d. upper limit

$$|\hat{\tilde{d}}^{\mathrm{Z}}_{\mu}| < 0.021 \quad \text{or} \quad |\tilde{d}^{\mathrm{Z}}_{\mu}| < 1.1 \cdot 10^{-17} e\mathrm{cm}, \tag{13}$$

while $\hat{f}_{\mathrm{V}\ell}$ and $\hat{f}_{\mathrm{A}\ell}$ are restricted from (12) at the 2 s.d. level by

$$\begin{aligned} \hat{f}^2_{\mathrm{V}\mu} + \hat{f}^2_{\mathrm{A}\mu} &< (2.6)^2, \\ \hat{f}^2_{\mathrm{V}\tau} + \hat{f}^2_{\mathrm{A}\tau} &< (2.6)^2. \end{aligned} \tag{14}$$

This type of reasoning to obtain limits for the dipole moments was frequently used (cf. e.g. [4,8]). The estimate for $\hat{\tilde{d}}^{\mathrm{Z}}_{\mu}$ in (13) updates the one given in [4].

For the WDM of the τ lepton it is not necessary to rely on indirect limits since experimental limits exist, obtained from direct CP violation searches. We quote the latest result for $|\mathrm{Re}d^{\mathrm{Z}}_{\tau}|$ which is related to our $\tilde{d}^{Z}_{\tau}$ as explained above: The combination of all LEP results [21] gives the 95% c.l. upper limits

$$|\hat{d}^{Z}_{\tau}| < 0.007 \quad \text{or} \quad |\mathrm{Re}d^{\mathrm{Z}}_{\tau}| \ < \ 3.6 \cdot 10^{-18} e\mathrm{cm}. \tag{15}$$

Estimates of the EDMs which contribute at tree level to radiative events only (see Fig. 1) can be given in a similar way by exploiting the experimentally allowed range between the SM prediction and the measured values of $\Gamma(\mathrm{Z} \to \ell^+\ell^-\gamma)$. An upper limit was given for the τ EDM in [22]. Using the Z decay data collected up to now it will be possible to update that value. The EDM of the muon is known to be smaller than $10^{-18} e$cm, or $|\hat{\tilde{d}}^{\gamma}_{\mu}| < 0.0019$ [20]. (Again, we can identify the on-shell EDM with the corresponding coupling parameter in (2) for our purposes.) The momentum correlations proposed in the following section are blind to the EDM, therefore our analysis of $\mathrm{Z} \to \mu^+\mu^-\gamma$ aims at only three still unknown parameters $f_{\mathrm{V}\mu}$, $f_{\mathrm{A}\mu}$ and $\tilde{d}^{\mathrm{Z}}_{\mu}$.

The calculation of the differential cross section shows that it is convenient to choose instead of the parameters $f_{\mathrm{V}\ell}$ and $f_{\mathrm{A}\ell}$ the linear combinations

$$\begin{aligned} \hat{f}_{1\ell} &= g_{\mathrm{V}\ell} \cdot \hat{f}_{\mathrm{A}\ell} - g_{\mathrm{A}\ell} \cdot \hat{f}_{\mathrm{V}\ell}, \\ \hat{f}_{2\ell} &= g_{\mathrm{V}\ell} \cdot \hat{f}_{\mathrm{V}\ell} - g_{\mathrm{A}\ell} \cdot \hat{f}_{\mathrm{A}\ell}, \end{aligned} \tag{16}$$

where $g_{\mathrm{V}\ell} = T_{3\ell} - 2Q_\ell \sin^2\theta_W$ and $g_{\mathrm{A}\ell} = T_{3\ell} = -1/2$ are the usual neutral current coupling constants of the leptons $\ell = \mu, \tau$. In the zero–mass limit $\hat{f}_{1\ell}$ induces an angular distribution of the $\ell^+\ell^-\gamma$ momenta which is measurable with tensor observables, while $\hat{f}_{2\ell}$ is related to vector observables. Thus, with appropriate observables these couplings can easily be determined independently.

3 Observables

In the following we consider the case of unpolarized $\mathrm{e}^+\mathrm{e}^-$ beams in the reactions (1). Then the initial state is described in the c.m. system by a CP–invariant density matrix (CP tests with longitudinally polarized beams have been discussed in [18,23]). Furthermore we assume that the experimental phase space cuts and the momentum reconstruction methods do not introduce a CP bias. Then the mean values of CP–odd momentum correlations trace the couplings defined in $\mathcal{L}_{\mathrm{CP}}$.

The task is now to construct a set of momentum correlations whose distributions are sensitive to the couplings defined in $\mathcal{L}_{\mathrm{CP}}$ with a reasonable statistical significance. For simplicity, we first consider each of the parameters $\hat{f_{1\ell}}$, $\hat{f_{2\ell}}$, $\hat{\tilde{d}}^{\mathrm{Z}}_{\ell}$ and $\hat{\tilde{d}}^{\gamma}_{\ell}$ separately by setting the other three to zero. Then the linear contribution to the mean value of a given observable, together with the width of its distribution, yields an estimate of the statistical accuracy with which this coupling can be measured.

The kinematics of the reaction $\mathrm{Z} \to \mu^+\mu^-\gamma$ is determined by the μ^+, μ^- momenta $\mathbf{k}_{\pm}$ in the Z rest frame which is to a good approximation the laboratory system at LEP and SLC. In the case of $\mathrm{Z} \to \tau^+\tau^-\gamma$ with subsequent one–prong τ decays which are the most frequent ones (about 85 %), the measurable quantities are primarily the momenta of the charged $\tau^{\pm}$ decay products, again denoted by $\mathbf{k}_{\pm}$, and the photon momentum $\mathbf{q}$. Using these momenta one can construct many CP-odd observables. We have calculated the signal–to–noise ratios of some basic correlations and found the following observables especially useful:

$$\begin{aligned} T &= (\hat{\mathbf{k}}_+ - \hat{\mathbf{k}}_-) \cdot \hat{\mathbf{p}}_+ \ (\hat{\mathbf{k}}_+ \times \hat{\mathbf{k}}_-) \cdot \hat{\mathbf{p}}_+, \\ V &= (\hat{\mathbf{k}}_+ \times \hat{\mathbf{k}}_-) \cdot \hat{\mathbf{p}}_+. \end{aligned} \tag{17}$$

Here hats denote unit momenta, and $\mathbf{p}_+$ is the initial positron momentum vector. With respect to the final state momenta T is a component of a tensor and V a component of a vector observable. These correlations involve neither particle energies nor the photon momentum. For $\mathrm{Z} \to \tau^+\tau^-\gamma$ we additionally studied the observables T^* and V^* which are defined as above but with $\hat{\mathbf{k}}_\pm$ taken in the rest system of the $\tau^+\tau^-$ pair. The τ momenta usually cannot be fully reconstructed from the observed decay products and the τ decay vertex information. But the observation of the photon should be sufficient to reconstruct in each event the $\tau^+\tau^-$ center of mass frame.

Let us denote the coupling parameters $\hat{f}_{1\ell}, \hat{f}_{2\ell}, \hat{d}_\ell^Z, \hat{d}_\ell^\gamma$ (cf. (2-4, 16)) by $g_{1\ell}, ..., g_{4\ell}$. The mean values of CP–odd observables such as T and V defined above are in general not purely linear functions of the $g_{i\ell}$ since the normalization introduces a denominator with quadratic contributions in the $g_{i\ell}$'s. The second order terms certainly are negligible if the couplings are small. For practical purposes it might be useful to consider observables which get *linear contributions only*. These can easily be obtained by multiplying any of the momentum correlations discussed in this article with the measured width $\Gamma(Z \to \ell^+\ell^-\gamma)$ for the phase space region, i.e. the cuts considered. In this way one constructs observables whose mean values are strictly linear in the couplings to be measured:

$$\Gamma(\mathrm{Z} \to \ell^+\ell^-\gamma) \quad \cdot \quad \langle \mathcal{O} \rangle \quad = \quad \sum_{i=1}^{4} a_i \cdot g_{i\ell}. \tag{18}$$

The a_i are proportionality constants which can be calculated numerically, and $\mathcal{O}$ denotes any CP–odd observable which does not depend on the unknown couplings.

In order to get an impression about the quality of the observables introduced above we also investigate for the couplings $g_{i\ell}$ the corresponding *optimal observables* which have the greatest possible statistical signal–to–noise ratio. For a single coupling this method was introduced in [24, 25]. The generalization to an arbitrary number of new couplings was given in [26] (cf. also [27]). The optimal observables are constructed as follows: Let us write the differential cross section as

$$\mathrm{d}\sigma(\phi) = \mathrm{d}\sigma_{\mathrm{SM}}(\phi) + \sum_{i=1}^{4} \mathrm{d}\sigma_i^{(1)}(\phi) g_{i\ell} + \sum_{i,j=1}^{4} \mathrm{d}\sigma_{ij}^{(2)}(\phi) g_{i\ell} g_{j\ell}. \tag{19}$$

Here ϕ stands collectively for the phase space variables, $\mathrm{d}\sigma_{\mathrm{SM}}$ denotes the CP–conserving SM part of the differential cross section, and $\mathrm{d}\sigma_i^{(1)}$ are the contributions due to the interference of the CP–odd interactions of $\mathcal{L}_{\mathrm{CP}}$ with the SM amplitudes. Then the ratios

$$\mathcal{O}_i(\phi) = \frac{\mathrm{d}\sigma_i^{(1)}(\phi)}{\mathrm{d}\sigma_{\mathrm{SM}}(\phi)}, \qquad (1 \le i \le 4) \tag{20}$$

are the observables with optimal statistical sensitivity for a joint measurement of the couplings assuming that the couplings which shall be measured are small. The observables $\mathcal{O}_i$ (20) are optimal for arbitrary phase space cuts. Their sensitivity, however, depends on the cuts.

For the reaction $\mathrm{Z} \to \mu^+\mu^-\gamma$ one can easily measure the momenta of all final state particles. Let q be the γ momentum and $k_{\bar{\mu}} \equiv k_+, k_\mu \equiv k_-$ the momenta of $\mu_\pm$. Thus in this case the phase space variable ϕ in (20) is given by

$$\phi \equiv (\mathbf{k}_+, \mathbf{k}_-, \mathbf{q}). \tag{21}$$

Due to the energy–momentum conservation constraint the phase space is 5–dimensional. Identification of a photon requires some isolation cuts. For our analysis we assume covariant cuts of the form

$$\frac{(k_{(1)} + k_{(2)})^2}{m_{\mathrm{Z}}^2} \ge y, \tag{22}$$

where $k_{(1)}, k_{(2)}$ are any two different four momenta from the set k_+, k_-, q, and y is chosen to be 0.03.

For the reaction $\mathrm{Z} \to \tau^+\tau^-\gamma$ with subsequent 1–prong τ decays

$$\mathrm{e}^+(p_+) + \mathrm{e}^-(p_-) \to \mathrm{Z} \to \tau^+(k_{\bar{\tau}}) + \tau^-(k_\tau) + \gamma(q), \tag{23}$$

$$\begin{aligned} &\tau^+(k_{\bar{\tau}}) \to a^+(k_+) + \text{neutrals}, \\ &\tau^-(k_\tau) \to b^-(k_-) + \text{neutrals}, \quad (a, b = \pi, \rho, \mathrm{e}, \mu) \end{aligned} \tag{24}$$

we assume integration over the phase space variables of the neutrals. We are then left ideally with the information from the momenta of $\tau^+, \tau^-, \gamma, a^+, b^-$ and thus here we can identify

$$\phi \equiv (\mathbf{k}_{\bar{\tau}}, \mathbf{k}_\tau, \mathbf{k}_+, \mathbf{k}_-, \mathbf{q}). \tag{25}$$

Of course, these momenta are not totally independent. Energy–momentum conservation in production and in the decays imposes well known constraints. Depending on the decay channels (24) the phase space is 9 to 11 dimensional. For our analysis below we assume again the isolation cuts (22) for the momenta of the γ and the charged decay products a^+, b^-. In practice the complete information on the $\tau^\pm$ momenta may not always be available. We will, therefore, discuss the optimal observables for the ideal case where the $\tau^\pm$ momenta can be reconstructed. This then gives the ideal sensitivities. On the other hand, for the simple observables T, V of (17) and their analogues T^*, V^* constructed in the $\tau^+\tau^-$ c.m. system one does not need knowledge of the individual τ^+ and τ^- momenta. Obviously, with partial information on the $\tau^\pm$ momenta one can then reach sensitivities in between.

From (20) we get as many optimal observables as we have parameters. Of course, in general the expectation values $\langle \mathcal{O}_i \rangle$ will get linear contributions from all four parameters in leading order. We have investigated the matrix describing this dependence of the mean values of observables on the parameters. For the optimal observables this is in leading order identical to the correlation matrix of the $\mathcal{O}_i$ (cf. [26]). We found that these correlations are either negligible, or the couplings contributing are bounded by independent experiments. In the following we therefore confine ourselves to discuss the diagonal elements of the covariance matrix. We calculated numerically the mean values and the widths of the $\mathcal{O}_i$ and got in this way for a given number of events the statistical accuracy with which each of the unknown parameters can be measured ideally, assuming that all other new couplings are zero. For a numerical evaluation it is not necessary to have the $\mathcal{O}_i(\phi)$ in explicit form. We use a Monte Carlo event generator[2] for the reactions (1) which takes into account the effects of $\mathcal{L}_{\rm CP}$. Since the optimal observables are composed from terms of the differential cross section, one can easily generate them numerically with a Monte Carlo method.

The knowledge of the optimal observables is interesting by itself, and useful for deriving simpler correlations which are approximately optimal. In most cases we found such observables which presumably are preferable for an experimental analysis.

Table 2. The variances $c_{ii} = \langle \mathcal{O}_i \mathcal{O}_i \rangle_{\rm SM}$ and the rescaled covariance matrix $\langle \mathcal{O}_i \mathcal{O}_j \rangle_{\rm SM} (c_{ii} c_{jj})^{-1/2}$ of the optimal observables for the measurement of the parameters $\hat{f}_{1,2\mu}$ and $\hat{\tilde{d}}{}^{\rm Z}_{\mu}$

c_{11}	c_{22}	c_{33}
$2.64 \cdot 10^{-3}$	$5.61 \cdot 10^{-4}$	$1.17 \cdot 10^{-8}$

	$\hat{f}_{1\mu}$	$\hat{f}_{2\mu}$	$\hat{\tilde{d}}{}^{\rm Z}_{\mu}$
$\hat{f}_{1\mu}$	1	0.000	0.000
$\hat{f}_{2\mu}$	0.000	1	-0.023
$\hat{\tilde{d}}{}^{\rm Z}_{\mu}$	0.000	-0.023	1

4 Z decays to $\mu^+\mu^-\gamma$

The LEP experiments have collected several thousand events of the reaction $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-\gamma$. For the CP tests proposed here it is sufficient to measure the muon directions of flight $\hat{\mathbf{k}}_\pm$. Using these one can calculate the mean values of the observables T and V defined in (17). The tensor correlation T gets a contribution only from $\hat{f}_{1\mu}$, allowing a clean determination of this parameter. On the other hand, the mean value of the vector observable V depends both on $\hat{f}_{2\mu}$ and $\tilde{d}^{\rm Z}_{\mu}$. As expected, the vector correlation is slightly less sensitive than the tensor observable, due to the smallness of the vector polarization of the Z boson. The contribution of the weak dipole moment to V is negligible: in order to cause a measurable effect it would have to be three orders of magnitude bigger than the indirect upper limit (13). We conclude that $\hat{f}_{2\mu}$ can be measured practically without contamination by other CP–violating sources using the observable V of (17).

We also investigated the correlation of a measurement of our parameters with optimal observables. Since the EDM coupling $\hat{d}^{\gamma}_{\mu}$ does not contribute to CP-odd terms in leading order (cf. [2]), we have here three parameters $\hat{f}_{1\mu}, \hat{f}_{2\mu}, \hat{d}^{Z}_{\mu}$ (denoted by $g_{1\mu}, ..., g_{3\mu}$) and three observables $\mathcal{O}_i$ (cf. (20)). Working always in leading order in the $g_{i\mu}$, we have

$$\langle \mathcal{O}_i \rangle = \sum_{j=1}^{3} c_{ij} \cdot g_{j\mu} \tag{26}$$

with

$$c_{ij} = \langle \mathcal{O}_i \mathcal{O}_j \rangle_{\rm SM}. \tag{27}$$

We calculated this covariance matrix. In Table 2 we give the c_{ii} (i=1,2,3) and the rescaled matrix

$$\langle \mathcal{O}_i \mathcal{O}_j \rangle_{\rm SM} / \sqrt{c_{ii} c_{jj}}, \tag{28}$$

where the diagonal elements are equal to 1. From these numbers we can easily derive parameters g'_i ($i = 1, 2, 3$) where the corresponding optimal observables $\mathcal{O}'_i$ have a diagonal covariance matrix $c'_{ij} = \langle \mathcal{O}'_i \mathcal{O}'_j \rangle_{\rm SM}$. These parameters can be chosen as

$$\begin{aligned} g'_1 &= \hat{f}_{1\mu}, \\ g'_2 &= \hat{f}_{2\mu} - 1.1 \cdot 10^{-4} \hat{\tilde{d}}{}^{\rm Z}_{\mu}, \\ g'_3 &= \hat{\tilde{d}}{}^{\rm Z}_{\mu}. \end{aligned} \tag{29}$$

Recall that for optimal observables the covariance matrix $V(g')$ of the estimated couplings is the inverse of the number of events times the covariance matrix of the observables (cf. [26,27])

$$V(g') = (Nc')^{-1}. \tag{30}$$

Thus a diagonal c' ensures a diagonal $V(g')$. We conclude from (29) that also for optimal observables the correlations are negligible. For a general discussion of diagonalization procedures in the estimation of more than one coupling parameter we refer to [27].

From the above we find that we have to consider for any of our observables $\mathcal{O} = T, V$ or an optimal one only a dependence on one coupling parameter. Calling it generically g we get in linear approximation

$$\langle \mathcal{O} \rangle = c \cdot g. \tag{31}$$

The 1 s.d. accuracy obtainable for a determination of g by a measurement of $\mathcal{O}$ can be estimated as

$$\delta g = \frac{\sqrt{\langle \mathcal{O}^2 \rangle_{\rm SM}}}{|c| \sqrt{N}}, \tag{32}$$

where $\langle \mathcal{O}^2 \rangle_{\rm SM}$ is the variance of $\mathcal{O}$, calculated in the SM, and N is the number of events within the cuts considered.

In Table 3 we summarize the statistical accuracies δg of (32) with which the WDM of the muon, $\hat{f}_{1\mu}$ and $\hat{f}_{2\mu}$ can be measured assuming a sample of $N = 10^4$ $\mu^+\mu^-\gamma$ events within the cut (22). The numbers in Table 3 represent the minimal sizes of the couplings which lead to visible effects at 1 s.d., or, if no deviation from the CP symmetry is found, δg is the 1 s.d. upper limit on the coupling parameter. The upper limits on $\hat{f}_{1\mu}, \hat{f}_{2\mu}$ correspond to two stripes in the $\hat{f}_{\rm V\mu}, \hat{f}_{\rm A\mu}$ parameter space. These are

[2] Monte Carlo event generators for the reactions $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-\gamma$ and $\mathrm{e}^+\mathrm{e}^- \to \tau^+\tau^-\gamma$ with subsequent τ decays are available from the authors

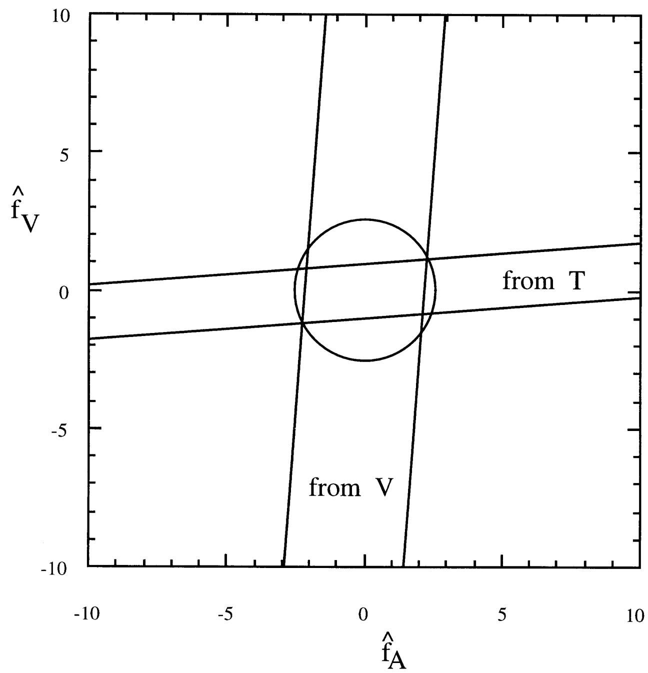

Fig. 2. Contour plot of the 2 s.d. errors on $\hat{f}_{V\mu}, \hat{f}_{A\mu}$ obtainable ideally from the measurement of the correlations T, V from Table 3 (*stripes*) and from the contribution to the width (*circle*) (c.f. (11,14)) assuming 10000 $\mu^+\mu^-\gamma$ events and zero mean values

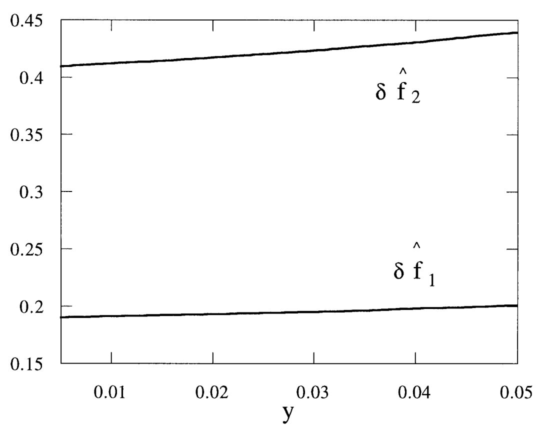

Fig. 3. 1 s.d. accuracies for the determination of $\hat{f}_{1\mu}$ and $\hat{f}_{2\mu}$ for different values of the phase space cut parameter y. Here we assume a number of $\mu^+\mu^-\gamma$ events corresponding to 10^4 for $y = 0.03$

depicted in Fig. 2, together with the indirect bound (14) assuming zero mean values and a phase space cut parameter $y = 0.03$. The sensitivities of the optimal observables for $\hat{f}_{1\mu}$ and $\hat{f}_{2\mu}$ are plotted as a function of the cut parameter y in Fig. 3.

We note that a search for CP–odd effects in the two–body decay $Z \to \mu^+\mu^-$ is in practice not possible [2] since the direction of flight of the muons carries no CP–odd information, and the spin analyzing muon decays usually escape detection. Nevertheless, this reaction should be useful to study possible detector effects which may feign

Table 3. 1 s.d. accuracies δg (32) with which the couplings $g = \hat{f}_{1\mu}$, $\hat{f}_{2\mu}$, and $\hat{\tilde{d}}{}^{Z}_{\mu}$ can be measured assuming a sample of 10000 decays $Z \to \mu^+\mu^-\gamma$ within the cut (22). Missing entries correspond to a vanishing sensitivity ($\delta g = \infty$)

	$\hat{f}_{1\mu}$	$\hat{f}_{2\mu}$	$\hat{\tilde{d}}{}^{Z}_{\mu}$
optimal observable	0.195	0.423	92.5
T	0.238		
V		0.523	101

non–zero mean values or asymmetric distributions of the correlations (17).

5 Z decays to $\tau^+\tau^-\gamma$

Compared to the muon case, CP tests in Z decays to $\tau^+\tau^-\gamma$ follow a somewhat different line: The τ momenta can not (yet) be measured directly. Instead we use the momenta of the charged decay products of the τ leptons to build the CP–odd correlations T, V and their counterparts T^*, V^* which use the momenta in the $\tau^+\tau^-$ rest frame. As explained in Sect. 3 we assume, however, knowledge of the $\tau^\pm$ momenta for the optimal observables.

In order to cover a large fraction (about 35%) of the $\tau^+\tau^-$ events, we investigate all combinations of the following one–prong decay channels:

$$\tau^\pm \to \nu\pi^\pm(k_\pm), \tag{33}$$

$$\to \nu\rho^\pm(k_\pm), \tag{34}$$

$$\to \nu\bar{\nu}\ell^\pm(k_\pm) \qquad (\ell = \mathrm{e}, \mu). \tag{35}$$

For these decays the SM matrix elements are assumed.

In Tables 4, 5 we collect the upper limits on the CP–odd couplings which can be obtained by measuring the optimal observables and the correlations T, V, T^*, and V^*. The numbers N of events for the various channels correspond roughly to a total number of 10^4 $Z \to \tau^+\tau^-\gamma$ events within the cut (22). Here we always assume that only one parameter $g = \hat{\tilde{d}}{}^{\gamma}_{\tau}, \hat{\tilde{d}}{}^{Z}_{\tau}, \hat{f}_{1\tau}, \hat{f}_{2\tau}$ is different from zero and quote δg calculated according to (31, 32) for the observable indicated. The measurement of the dipole moments relies essentially on the analysis of the τ spins. The momentum distribution of the final state particles depends on the τ polarization. For instance, the π^+ in the reaction (33) is emitted preferentially opposite to the τ^+ spin direction, if we consider the decay in the τ^+ rest system. The momentum correlations T, V, T^*, V^* which we use project on the CP–asymmetric part of the spin-spin-correlation of the τ pair. Therefore they are rather insensitive to the electric dipole coupling $\tilde{d}^{\gamma}_{\tau}$ which affects the polarization of only one τ lepton (see Fig. 1c). The weak dipole coupling $\tilde{d}^{Z}_{\tau}$, however, can be measured with an accuracy of the order of $10^{-17}e$cm assuming that 10^4 $Z \to \tau^+\tau^-\gamma$ decays are available. The obvious gap between the sensitivities of the optimal observables and the simple correlations originates

Table 4. 1 s.d. accuracies with which $\hat{d}^{\gamma}_{\tau}$ and $\hat{d}^{Z}_{\tau}$ can be measured for a given number N of events in various τ decay modes

		$\hat{d}^{\gamma}_{\tau}$					$\hat{d}^{Z}_{\tau}$				
channel	N	opt.	T	V	T^*	V^*	opt.	T	V	T^*	V^*
$\pi - \pi$	100	0.40	91	59	8.4	5.0	0.054	0.83	8.3	0.077	0.67
$\pi - \rho$	400	0.41	67	41	13	3.2	0.038	0.63	38	0.051	1.8
$\rho - \rho$	400	0.85	150	83	42	5.9	0.070	1.3	28	0.092	44
$\ell - \ell$	800	0.41	74	51	14	4.8	0.044	0.73	12	0.063	0.63
$\ell - \pi$	600	0.26	120	95	7.5	17	0.030	0.98	3.4	0.140	0.46
$\ell - \rho$	1200	1.5	1400	810	53	40	0.13	38	23	0.91	4.3
combined		**0.17**	40	26	4.8	**2.2**	**0.018**	0.37	3.0	**0.032**	0.32

Table 5. 1 s.d. accuracies with which $\hat{f}_{1\tau}$ and $\hat{f}_{2\tau}$ can be measured for a given number N of events in various τ decay modes

		$\hat{f}_{1\tau}$					$\hat{f}_{2\tau}$				
channel	N	opt.	T	V	T^*	V^*	opt.	T	V	T^*	V^*
$\pi - \pi$	100	0.61	1.7	7.1	104	45	1.1	3.4	3.7	36	31
$\pi - \rho$	400	0.53	0.85	5.9	59	32	0.88	2.9	1.9	38	36
$\rho - \rho$	400	0.70	0.88	17	60	57	1.3	8.6	2.0	46	68
$\ell - \ell$	800	0.48	0.60	4.8	29	62	0.87	2.2	1.4	22	34
$\ell - \pi$	600	0.39	0.66	7.2	20	700	0.68	3.2	1.5	23	140
$\ell - \rho$	1200	1.4	1.7	130	74	340	2.5	64	3.8	190	1200
combined		**0.23**	**0.34**	3.0	15	22	**0.4**	1.4	**0.78**	13	18

from the fact that the optimal observables as we calculate them work with the knowledge of the τ momentum. At least for the measurement of the electric dipole moment this seems to be a basic ingredient. Since a measurement of the EDM and WDM is based on the τ-spin information, we observe a rather strong dependence of the sensitivities in Table 4 on the τ-decay channels, corresponding to their spin-analyzing qualities.

The parameters $\hat{f}_{1\tau}$, $\hat{f}_{2\tau}$ can be determined with an accuracy of 1 s.d. when they are of the order of about 0.5 (Table 5), given the assumed number of events. Here it should be advantageous to use the observables T and V which get only tiny contributions from the dipole moments of the tau lepton. These results are comparable to the muon case (Table 3). It is worth noting that the sensitivities are more or less independent of the tau decay channel: the measurement of $\hat{f}_{1\tau}, \hat{f}_{2\tau}$ is not substantially based on the spin analyzing qualities of the different decay modes.

6 Conclusions

In this article we have investigated the radiative decays $Z \to \ell^+\ell^-\gamma$ ($\ell = \mu, \tau$) with respect to the possibility of performing tests of the CP symmetry. The relevant CP-odd parameters are the EDM and WDM coupling parameters of the leptons, $\tilde{d}^{\gamma}_{\ell}, \tilde{d}^{Z}_{\ell}$ and 4-point coupling parameters $\hat{f}_{i\ell}$. For the muon case the EDM does not contribute to CP-odd correlations in Z decays to leading order. But in the effective Lagrangian approach $\tilde{d}^{\gamma}_{\mu}$ can be bounded by the limits obtained in the direct search for an EDM of the muon. On the other hand, the radiative Z decays offer the possibility to measure the muon's WDM and its 4-point couplings $\hat{f}_{1\mu}, \hat{f}_{2\mu}$. The accuracies obtainable with a realistic number of events are collected in Table 3.

For the τ-lepton one can in principle measure all four CP-odd parameters $\tilde{d}^{\gamma}_{\tau}, \tilde{d}^{Z}_{\tau}, \hat{f}_{1\tau}, \hat{f}_{2\tau}$ using radiative Z decays. The measurement of $\hat{d}^{Z}_{\tau}$ using about 10^4 $Z \to \tau^+\tau^-\gamma$ decays has a 1 s.d. sensitivity (cf. Table 4)

$$\delta\hat{d}^{Z}_{\tau} = 0.018. \tag{36}$$

This is, as expected, weaker than the bound (15) obtained from $Z \to \tau^+\tau^-$ decays. Radiative Z decays offer the possibility to perform a true measurement of the EDM coupling parameter $\hat{d}^{\gamma}_{\tau}$. To date no direct determination of $\hat{d}^{\gamma}_{\tau}$ exists. Another reaction where $\hat{d}^{\gamma}_{\tau}$ can be measured is $e^+e^- \to \tau^+\tau^-$ away from the Z resonance (cf. [7]). With 10^6 τ-pairs at $\sqrt{s} = 10$ GeV for instance one gets a sensitivity:

$$\delta \hat{d}^{\gamma}_{\tau} = 0.043. \tag{37}$$

This is to be compared to $\delta \hat{d}^{\gamma}_{\tau} = 0.17$ or 2.2 in Table 4 using the optimal observable or V^*, respectively. Finally, the parameters $\hat{f}_{1\tau}, \hat{f}_{2\tau}$ are special to $Z \to \tau^+\tau^-\gamma$ decays and can be measured with accuracies of order 0.5 with the number of events given in Table 5.

An investigation of radiative Z decays can thus give information of CP-odd coupling parameters of leptons which is complementary to the one obtainable from other sources. We have found that for the observables and cuts we considered correlations between the CP–odd coupling parameters are negligible. Thus we investigated each coupling parameter separately. In an analysis of real experimental data other cuts may be used and smearing due to detector effects will play a role. One will again be confronted with the question of correlations and in a sophisticated analysis one will have to consider the parameters $\hat{f}_{1\ell}, \hat{f}_{2\ell}, \hat{d}^{\gamma}_{\ell}, \hat{d}^{Z}_{\ell}$ simultaneously for each lepton ℓ. It should then be advantageous to use couplings and observables which give diagonal covariance matrices. The general methods to achieve this which are explained in [27] can easily be adapted for our case here.

Independently of CP-violation it should be interesting to study the reaction $\mathrm{Z} \to \ell^+\ell^-\gamma$ in the phase space region where the photon is "hard". Any new effects in lepton physics, like substructures or excited leptons, could lead to contact interactions between Z, ℓ and γ. These should best show up in "hard" 3-body decays, i.e. in regions of phase space where the photon has high energy and is well separated from ℓ^+ and ℓ^-. There the SM contribution is relatively small. An investigation of the ratio

$$R(y) = \frac{\Gamma(\mathrm{Z} \to \ell^+\ell^-\gamma)}{\Gamma_{\mathrm{SM}}(\mathrm{Z} \to \ell^+\ell^-\gamma)} \tag{38}$$

as function of the cut parameter y (22) should already be revealing. As an example we have calculated this ratio for our ansatz SM plus $\mathcal{L}_{\mathrm{CP}}$ (2) assuming that only $f_{\mathrm{V}\ell}$ and $f_{\mathrm{A}\ell}$ are different from zero. To a very good approximation (cf. [9]) these couplings add to $\Gamma(\mathrm{Z} \to \ell^+\ell^-)$ a term $\Delta\Gamma_{\mathrm{CP}}(f_{\mathrm{V}\ell}, f_{\mathrm{A}\ell}; y)$ proportional to $\hat{f}^2_{\mathrm{V}\ell} + \hat{f}^2_{\mathrm{A}\ell}$. Thus we plot

$$\frac{R(y) - 1}{\hat{f}^2_{\mathrm{V}\ell} + \hat{f}^2_{\mathrm{A}\ell}} = \frac{\Delta\Gamma_{\mathrm{CP}}(\hat{f}_{\mathrm{V}\ell}, \hat{f}_{\mathrm{A}\ell}; y)}{\Gamma_{\mathrm{SM}}(\mathrm{Z} \to \ell^+\ell^-\gamma)(\hat{\mathrm{f}}^2_{\mathrm{V}\ell} + \hat{\mathrm{f}}^2_{\mathrm{A}\ell})} \tag{39}$$

in Fig. 4 (solid line). We see that a value $\hat{f}^2_{\mathrm{V}\ell} + \hat{f}^2_{\mathrm{A}\ell} = 1$ will lead to deviations of $R(y)$ from 1 of up to about 10% at $y = 0.2$. An investigation of

$$r(y) = \frac{\Delta\Gamma_{\mathrm{CP}}(\hat{f}_{\mathrm{V}\ell}, \hat{f}_{\mathrm{A}\ell}; y)}{\Delta\Gamma_{\mathrm{CP}}(\hat{f}_{\mathrm{V}\ell}, \hat{f}_{\mathrm{A}\ell})}, \tag{40}$$

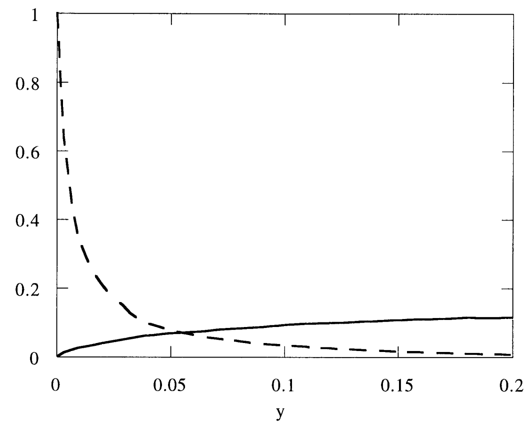

Fig. 4. The ratio $(R(y) - 1)/(\hat{f}^2_{\mathrm{V}\ell} + \hat{f}^2_{\mathrm{A}\ell})$ (*solid line*) as given in (38, 39), and the ratio $r(y)$ (*dashed line*) defined in (40) as function of the cut parameter y (22)

plotted in Fig. 4 (dashed line) shows that the *absolute* contribution of CP–odd parameters to the width decreases rapidly for increasing values of the cut parameter y. Here $\Delta\Gamma_{\mathrm{CP}}(\hat{f}_{\mathrm{V}\ell}, \hat{f}_{\mathrm{A}\ell})$ is the addition to the width for cut parameter $y = 0$ as given in (10).

The calculations reported in this article required a considerable amount of symbolic and numerical computation. We used the new programming language M [28] for the symbolic manipulations including the expansion of traces of Dirac–matrices, simplification of the resulting large expressions, and generation of optimized Fortran program code. The numerical phase space integrations (with dimensions up to 11) were done with the Fortran routine VEGAS. The programs which we used as well as Monte Carlo event generators for the reactions discussed in this article can be obtained from the authors (World Wide Web address: `http://www.thphys.uni-heidelberg/~overmann`).

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References

1. J.F.Donoghue, B.R.Holstein, G.Valencia: Int.J.Mod.Phys. **A2**, 319 (1987)
2. W.Bernreuther, U.Löw, J.P.Ma, O.Nachtmann: Z.Phys. **C43**, 117 (1989)
3. W.Bernreuther, O.Nachtmann: Phys.Rev.Lett **63**, 2787 (1989), E: **64**, 1072 (1990)
4. J.Körner, J.P.Ma, R.Münch, O.Nachtmann, R.Schöpf: Z. Phys. **C49**, 447 (1991)
5. W. Bernreuther, G.W. Botz, O.Nachtmann, P.Overmann: Z. Phys. **C52**, 567 (1991)
6. W.Bernreuther, O.Nachtmann: Phys.Lett. **B268**, 424 (1991)

7. W.Bernreuther, O.Nachtmann, P.Overmann: Phys.Rev. **D48**, 78 (1993)
8. W.Bernreuther, G.W.Botz, D.Bruß, P.Haberl, O.Nachtmann: Z.Phys. **C68**, 73 (1995)
9. P. Haberl: "CP violating couplings in Z → 3 jets revisited", eprint hep-ph/9611430
10. A.De Rujula, M.B.Gavela, O.Pene, F.J.Vegas: Nucl.Phys. **B357**, 311 (1991)
11. K.J.Abraham, B.Lampe: Phys. Lett. **B326**, 175 (1994)
12. W.Bernreuther, O.Nachtmann: "Some Remarks on the Search for CP Violation in Z-Decays", eprint hep-ph/9603331 (1996), to be published in Z.Phys. **C**
13. P.D.Acton et al. (OPAL collab.): Phys.Lett. **B281**, 405(1992); R.Akers et al. (OPAL collab.): Z.Phys. **C66**, 31 (1995); K. Ackerstaff et al. (OPAL collab.): "Search for CP Violation in $Z^0 \to \tau^+\tau^-$ and an Upper Limit on the Weak Dipole Moment of the τ Lepton", preprint CERN-PPE/96-184 (1996)
14. D.Busculic et al. (ALEPH collab.): Phys.Lett. **B297**, 459 (1992);
    Phys.Lett. **B346**, 371 (1995)
15. A.Stahl: Nucl.Phys. **B** (Proc. Suppl.) **40**, 505 (1995)
16. D. Busculic et al (ALEPH collab.): Phys.Lett. **B384**, 365 (1996)
17. A.Venturi, in Proc. of the XXVII Int. Conf. on High Energy Physics (eds. P.J. Bussey, I.G. Knowles), Bristol (1995), p.771
18. W.Bernreuther, A.Brandenburg, P.Overmann, Phys.Lett. **B391**, 413 (1997)
19. The LEP Electroweak Working Group and the SLD Heavy Flavor Group: A Combination of Preliminary LEP and SLD Electroweak Measurements and Constraints on the Standard Model", Internal Note LEPEWWG/96-02 (1996)
20. Particle Data Group: Phys. Rev. **D54** (1996)
21. N. Wermes: "CP Tests and Dipole Moments in τ-Pair Production Experiments", Contribution to the International Workshop on Tau Lepton Physics (TAU96), Estes Park (1996); Univ. Bonn report BONN HE-90-10, ISSN-0172-8733
22. J.A.Grifols, A.Méndez: Phys.Lett. **B255**, 611 (1991)
23. B.Ananthanarayan, S.Rindani: Phys.Rev. **D52**, 2684 (1995); P.Poulose, S.Rindani: Phys.Rev. **D54**, 4326 (1996)
24. D.Atwood, A.Soni, Phys.Rev. **D45**, 2405 (1992)
25. M.Davier, L.Duflot, F.Le Diberder, A.Rougé: Phys.Lett. **B306**, 411 (1993)
26. M.Diehl, O.Nachtmann: Z.Phys. **C62**, 397 (1994)
27. M.Diehl, O.Nachtmann: "Anomalous three gauge boson couplings in $e^+e^- \to W^+W^-$ and "optimal" strategies for their measurement", eprint hep-ph/9702208, to appear in Eur. J. Phys. C
28. P.Overmann: "M – Reference Manual" (in preparation)